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ELECTRICAL ENGINEERING

CONVENTIONAL PRACTICE SETS VOLUME - II

CONTENTS

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CONTROL SYSTEMS

CONVENTIONAL PRACTICE SETS

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1 CHAPTER Introduction

Q.1 (a) A control system is defined by following mathematical relationship

$$\frac{d^2x}{dt^2} + \frac{6dx}{dt} + 5x = 12(1 - e^{-2t})$$

Find the response of the system at $t \to \infty$

(b) A function $y(t)$ satisfies the following differential equation

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

Where $\delta(t)$ is delta function. Assuming zero initial condition and denoting unit step function by $u(t)$. Find $y(t)$.

Solution:

(a) Taking LT on both sides

$$(s^2 + 6s + 5)\, X(s) = 12\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$(s + 1)\,(s + 5)\, X(s) = \frac{24}{s(s+2)}$$

$$X(s) = \frac{24}{s(s+1)\,(s+2)\,(s+5)}$$

Response at $t \to \infty$

Using final value theorem,

$$\boxed{\lim_{t \to \infty} x(t) = \lim_{s \to 0} [sX(s)]} = \lim_{s \to 0} \frac{s \times 24}{s(s+1)\,(s+2)\,(s+5)} = 2.4$$

(b) Taking Laplace transform on both sides

$$Y(s)\,[s + 1] = 1$$

$$Y(s) = \frac{1}{s+1}$$

By taking inverse Laplace transform

$$y(t) = e^{-t}\, u(t)$$

Q.2 (a) The Laplace equation for the charging current, $i(t)$ of a capacitor arranged in series with a resistance is given by

$$I(s) = \frac{sC}{1 + sRC} \cdot E(s)$$

The circuit is connected to a supply voltage of E. If E = 100 V, R = 2 MΩ, C = 1 μF. Calculate the initial value of the charging current.

(b) A series circuit consisting of resistance R and an inductance of L is connected to a d.c. supply voltage of E. Derive an expression for the steady-state value of the current flowing in the circuit using final value theorem.

Solution:

(a) Since, $E = 100\, v(t)$

Taking Laplace Transform, $E = 100\,(t)$ volts,

$$\therefore \quad E(s) = \frac{100}{s}$$

Substituting the given values,

$$I(s) = \frac{1\times 10^{-6} s}{(2\times 10^{6}\times 1\times 10^{-6} s + 1)}\cdot\frac{100}{s} = \frac{10^{-6} s}{2s+1}\cdot\frac{100}{s}$$

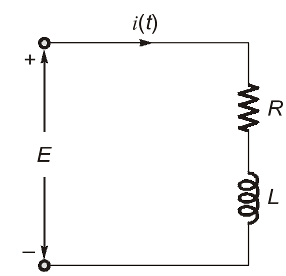
Applying the initial value theorem,

$$\boxed{i(0^{+}) = \lim_{t\to 0} i(t) = \lim_{s\to\infty} s\,I(s)}$$

$$i(0^{+}) = \lim_{s\to\infty} s\cdot\frac{10^{-4}}{1+2s} = \lim_{s\to\infty}\cdot\frac{10^{-4}}{\frac{1}{s}+2} = 50\ \mu\text{A}$$

(b) The differential equation relating the current $i(t)$ flowing in the circuit and the input voltage E is given by

$$E = R\,i(t) + L\frac{di(t)}{dt}$$



Taking Laplace transform of the equation yields,

$$E(s) = R\,I(s) + L[(sI(s) - i(0^{+}))]$$

Assume, $i(0^{+}) = 0$

$$\therefore \quad E(s) = R\,I(s) + Ls\,I(s)$$

$\because$ E is constant (d.c. voltage)

$$E(s) = \frac{E}{s} = RI(s) + Ls\,I(s)$$

$$I(s) = \frac{E}{s(R+sL)}$$

Applying the final value theorem,

$$i_{ss} = \lim_{t\to\infty} i(t) = \lim_{s\to 0} s\,I(s) = \lim_{s\to 0}\frac{sE}{s(R+sL)}$$

$$i_{ss} = \frac{E}{R}$$

Q.3 **The impulse response of a system S_1 is given by $y_1(t) = 4e^{-2t}$. The step response of a system S_2 is given by $y_2(t) = 2(1 - e^{-3t})$. The two systems are cascaded together without any interaction. Find response of the cascaded system for unit ramp input.**

Solution:

(a) Taking the Laplace transform of the response of S_1, we get

$$Y_1(s) = \frac{4}{s+2},$$

$$X_1(s) = 1 \ \dots (x(t) = \delta(t))$$

Therefore, $$G_1(s) = \frac{Y_1(s)}{X_1(s)} = \frac{4}{s+2} \qquad [\because Y_1(s) = 1]$$

Taking the Laplace transform of the response of S_2, we get

$$Y_2(s) = 2\left(\frac{1}{s} - \frac{1}{s+3}\right) = \frac{6}{s(s+3)}$$

$$Y_2(s) = \frac{1}{s} \quad \ldots (x_2(t) = u(t))$$

Thus, $$G_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{6}{s(s+3)} \cdot s = \frac{6}{s+3}$$

(b) The transfer function of the cascaded system is

$$G(s) = G_1(s)G_2(s) = \frac{24}{(s+2)(s+3)}$$

The Laplace transform of unit ramp is $R(s) = \frac{1}{s^2}$. Therefore,

$$G(s) = \frac{C(s)}{R(s)}$$

$$C(s) = \frac{24}{(s+2)(s+3)} \cdot \frac{1}{s^2}$$

$$\equiv \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$A = \left.\frac{24}{(s+2)(s+3)}\right|_{s=0} = 4$$

$$B = \frac{d}{ds}\left[s^2 C(s)\right]_{s=0}$$

$$= \frac{d}{ds}\left[\frac{24}{(s+2)(s+3)}\right] = -\left.\frac{24(2s+5)}{(s+2)^2(s+3)^2}\right|_{s=0}$$

$$= -\frac{10}{3}$$

$$C = \left.\frac{24}{s^2(s+3)}\right|_{s=-2} = 6$$

$$D = \left.\frac{24}{s^2(s+2)}\right|_{s=-3} = -\frac{8}{3}$$

$$C(s) = \frac{4}{s^2} - \frac{10}{3}s + \frac{6}{s+2} - \frac{8}{3}e^{-3t}$$

Taking inverse Laplace transform.

Therefore, $$c(t) = 4t - \frac{10}{3}u(t) + 6e^{-3t} - \frac{8}{3}e^{-3t}$$

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ANALOG ELECTRONICS

CONVENTIONAL PRACTICE SETS

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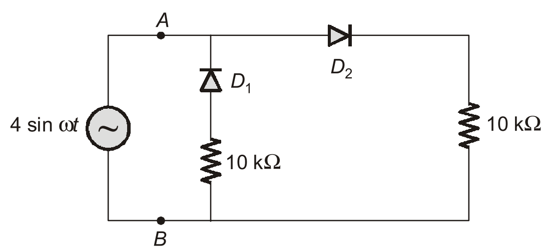
CHAPTER 1 Diode Circuits

Q.1 A voltage source $V_{AB} = 4 \sin \omega t$, is applied across the terminals A and B of the circuit. The diodes are assumed to be ideal. Find the impedance offered by the circuit across the terminals A and B in kilo ohm.

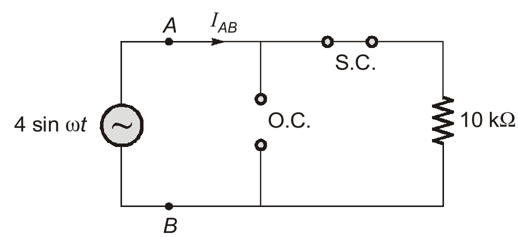


Solution:

In +ve half cycle D_1 – off (R.B.)

D_2 – on (F.B.)

∴ Equivalent circuit will be



∴ $$V_{AB} = 4 \sin\omega t$$

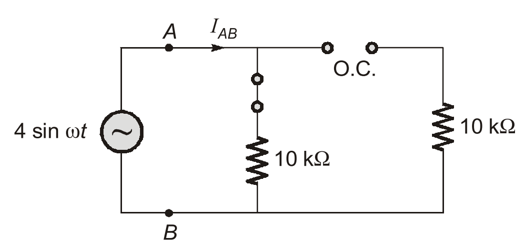
$$I_{AB} = \frac{V_{AB}}{10\,\text{k}\Omega}$$

∴ $$R_i = \frac{V_{AB}}{I_{AB}} = 10\,\text{k}\Omega$$

For –ve half cycle,

D_1 on, D_2 off

Equivalent circuit,

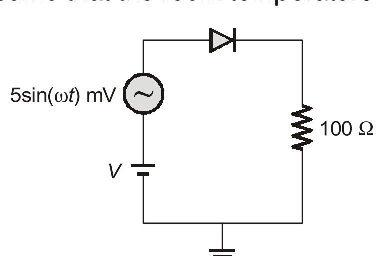


$$V_{AB} = 4 \sin\omega t$$

$$I_{AB} = \frac{4 \sin\omega t}{10\,\text{k}\Omega}$$

∴ $$\frac{V_{AB}}{I_{AB}} = R_i = 10\ \text{k}\Omega$$

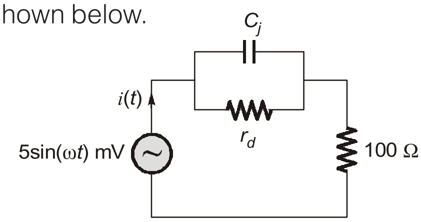
Q.2 A DC current of 26 μA flows through the circuit shown. The diode in the circuit is forward biased and it has an ideality factor of one. At the quiescent point, the diode has a junction capacitance of 0.5 nF. Its neutral region resistances can be neglected. Assume that the room temperature thermal equivalent voltage is 26 mV.



For $\omega = 2 \times 10^6$ rad/s, the amplitude of the small-signal component of diode current.

Solution:

The small-signal equivalent model of the given circuit can be drawn as shown below.



Given that,

$$\omega = 2 \times 10^6 \text{ rad/sec}$$
$$C_j = 0.5 \text{ nF}$$
$$I_{DC} = 26\ \mu\text{A}$$
$$V_T = 26 \text{ mV}$$
$$\eta = 1$$

Since, small signal incremental diode resistance, $r_d = \dfrac{\eta V_T}{I_{DC}} = \dfrac{26 \text{ mV}}{26\ \mu\text{A}} = 1\text{k}\Omega$

and impedance due to junction capacitance, $\dfrac{1}{\omega C_j} = \dfrac{1}{2\times10^6 \times 0.5\times10^{-9}}\ \Omega = 1\text{k}\Omega$

So, total impedance of the circuit will be,

$$Z = \left(r_d \| \frac{1}{j\omega C_j}\right) + 100\ \Omega$$

$$\left(r_d \| \frac{1}{j\omega C_j}\right) = \frac{(1000)(-j1000)}{1000 - j1000}\ \Omega = \frac{-j(1+j)}{2}\text{k}\Omega = \frac{1}{2}(1-j)\text{ k}\Omega = (500 - j500)\ \Omega$$

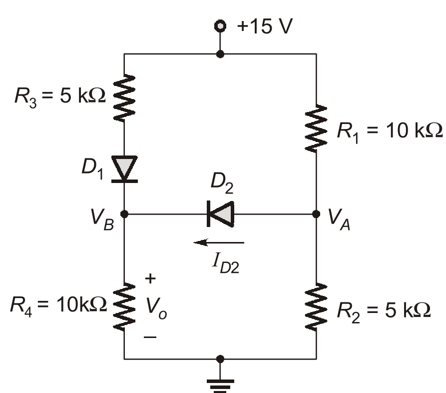
$\therefore$

$$Z = 600 - j500\ \Omega$$

$$|Z| = 100\sqrt{36+25} = 100\sqrt{61}\ \Omega$$

$$I_m = \frac{V_m}{|Z|} = \frac{5 \text{ mV}}{100\sqrt{61}\ \Omega} = \frac{50}{\sqrt{61}}\ \mu\text{A} = 6.40\ \mu\text{A}$$

Q.3 Determine the current I_{D2} and the voltage V_o in the multidiode circuit shown in the figure below. Assume that, cut-in voltage $V_\gamma = 0.7$ V for each diode.



Solution:

To begin, initially assume that, both the diodes D_1 and D_2 are in their conducting state.

By applying KCL at A and B nodes, we have

$$\frac{15 - V_A}{10} = I_{D2} + \frac{V_A}{5} \quad \text{...(i)}$$

and

$$\frac{15 - (V_B + 0.7)}{5} + I_{D2} = \frac{V_B}{10} \quad \text{...(ii)}$$

We note that $V_B = V_A - 0.7$. Combining the two equations and eliminating I_{D2}, we find

$$V_A = 7.62 \text{ V} \quad \text{and} \quad V_B = 6.92 \text{ V}$$

From equation (i) above, we obtain

$$\frac{15 - 7.62}{10} = I_{D2} + \frac{7.62}{5} \Rightarrow I_{D2} = -0.786 \text{ mA}$$

We assumed that D_2 was ON, so a negative current is inconsistent with that initial assumption.

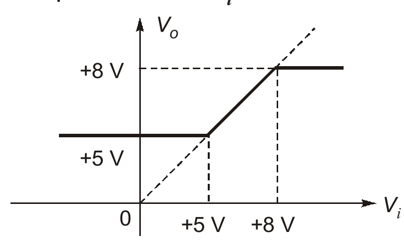
Now assume that diode D_2 is OFF and D_1 is ON. To find the node voltages, we can simply use voltage divider principle as

$$V_A = \left(\frac{5}{5+10}\right)(15) = 5\text{ V}$$

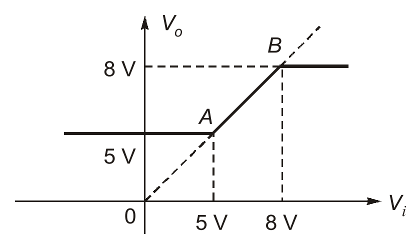
and
$$V_B = V_o = \left(\frac{10}{10+5}\right)(15-0.7) = 9.53\text{ V}$$

These voltages show that diode D_2 is indeed reverse biased so that $I_{D2} = 0$.

Q.4 The ideal transfer characteristic of a particular circuit is given in figure. Design the circuit. Draw the output waveform with proper explanation, if V_i = 10 sin ωt.



Solution:



Slope of the curve between A and B is

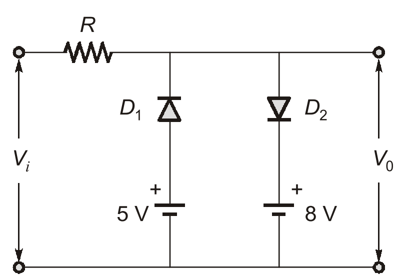
$$m = \frac{(8-5)}{(8-5)} = 1$$

The circuit diagram for the above input-output (transfer) characteristic is a two-level clipper as shown below.

Cut in voltages of diodes are zero.

For $V_i < 5$ V → diode D_1 will be on and D_2 will be off

and $\boxed{V_0 = 5\text{ V}}$



For $V_i > 8$ V → diode D_1 will be off and diode D_2 will be on

and $\boxed{V_0 = 8\text{ V}}$

For $5 < V_i < 8$ V → both the diodes will be off

and $\boxed{V_0 = V_i}$

Given that $V_i = 10\sin\omega t$

or $V_i = 10\sin\theta$ $(\omega t = \theta)$

For $V_i < 5$; $10\sin\theta < 5 \Rightarrow 0 < \theta < 30°$ and $150° < \theta < 360°$

$V_o = 5$ V

For $V_i > 8$ V; $10\sin\theta > 8 \Rightarrow 53.13° < \theta < 126.869°$

$V_o = 8$ V

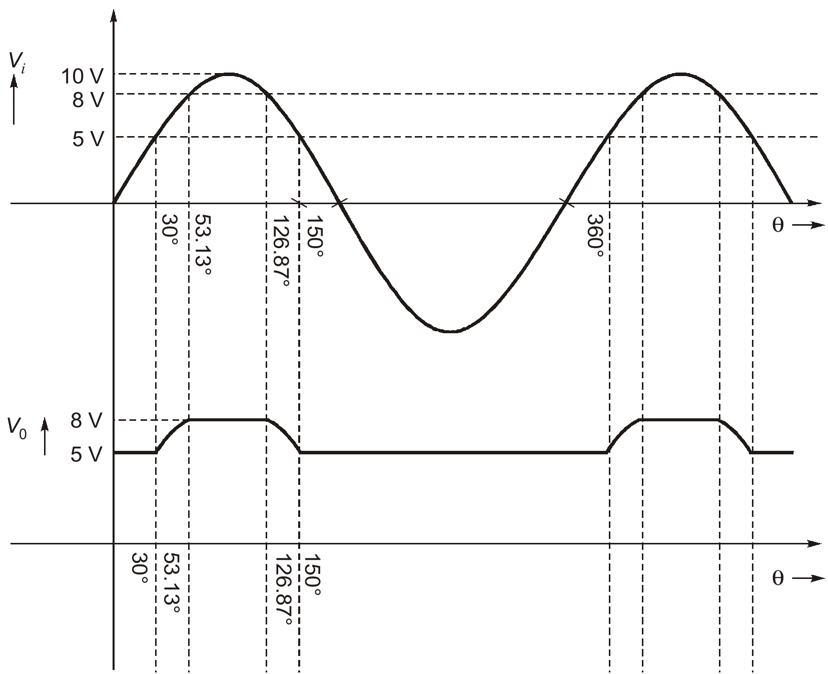
For $5 < V_i < 8$ V; $30° < \theta < 53.13°$ and $126.87° < \theta < 150°$

$V_o = V_i$

The required voltage-current characteristics can be written as

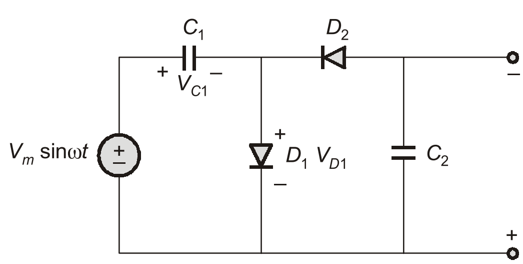
$$V_o = \begin{cases} 5\text{ V} & ; \quad V_i < 5\text{ V} \\ V_i & ; \quad 5\text{ V} < V_i < 8\text{ V} \\ 8\text{ V} & ; \quad V_i > 8\text{ V} \end{cases}$$

Now output waveform will be



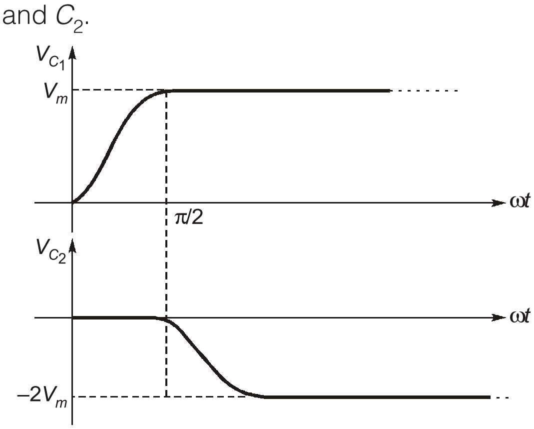
Q.5 Draw the neat circuit of a voltage doubler. Explain its operation. Draw the waveforms for the voltages across the two capacitors.

Solution:



Voltage follower circuit

The figure shows a circuit-composed of two sections in cascade, a clamp circuit formed by C_1 and D_1 and peak rectifier formed by D_2 and C_2.



When excited by a sinusoidal of amplitude V_m the clamping section provides the waveform shown. Assuming ideal diodes, while the positive peaks are damped to 0 V, the negative peak reaches $-2V_m$. In response to this wave form, the peak-detector section provides, across capacitor C_2 a negative dc voltage of magnitude $2V_m$. Because the output voltage is double the input peak, the circuit is known as a voltage doubler.

Q.6 For the circuit shown in the figure (a), the input voltage waveform V_i is shown in figure (b). Assume that the RC time constant is large and the cut-in voltage of diode $V_\gamma = 0$ V. Determine the output voltage waveform.

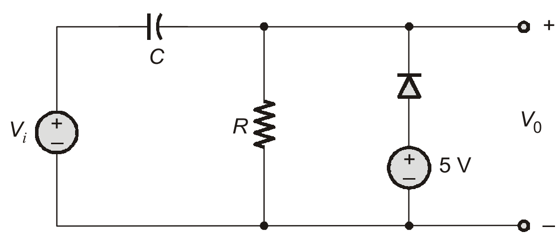


Fig. (a)

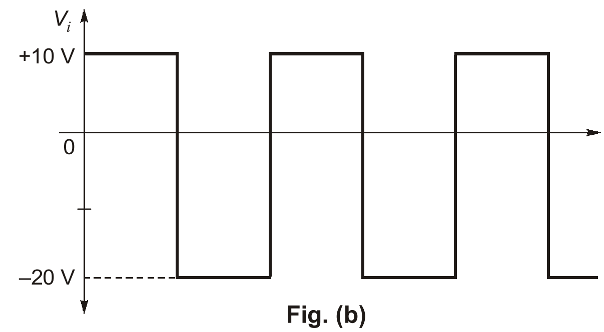
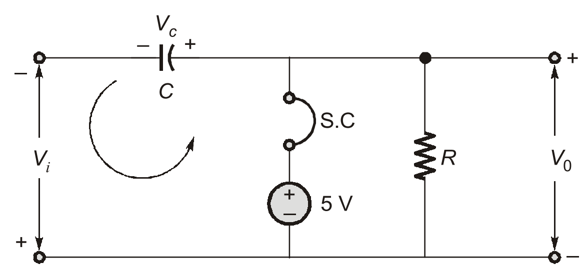


Fig. (b)

Solution:

Considering first the negative half cycle of the input signal V_i, the diode will be forward biased and acts as a short-circuit. The equivalent circuit can be drawn as:



The capacitor charges and the voltage across the capacitor can be calculated using KVL as below:

$$-V_i - 5 + V_c = 0$$

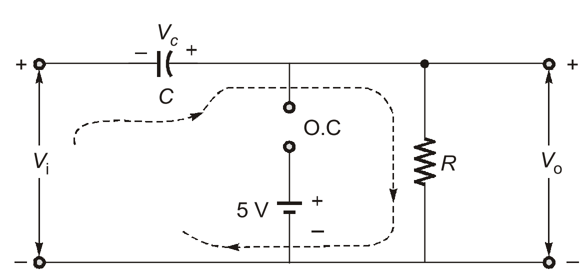
or $$V_c = V_i + 5 \quad \text{or} \quad V_c = 20 + 5$$

∴ $$V_c = 25 \text{ volt}$$

During this period, diode is short circuited, hence battery voltage (+5 V) appears across the output,

$$V_0 = 5 \text{ V}$$

Now, we have to consider the positive half cycle of the input voltage where the diode '*D*' is reversed biased and so it acts as open circuit. The capacitor voltage 'V_c' (due to charging in the previous negative half cycle duration) comes in the series as shown in the circuit given below:



$$-V_i - V_c + V_0 = 0$$

or $$V_0 = V_i + V_c \quad \text{or} \quad V_0 = 10 + 25$$

∴ $$V_0 = 35 \text{ V}$$

DIGITAL ELECTRONICS

CONVENTIONAL PRACTICE SETS

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1 CHAPTER Number Systems and Codes

Q.1 (i) Convert octal 756 to decimal.
(ii) Convert hexadecimal 3B2 to decimal.
(iii) Convert the long binary number 1001001101010001 to octal and to hexadecimal.

Solution:

(i) $(756)_8$

$$= 7 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 = 448 + 40 + 6 = (494)_{10}$$

(ii) $(3B2)_{16}$

$$= 3 \times 16^2 + 11 \times 16^1 + 2 \times 16^0 \qquad \text{(put B = 11)}$$

$$= 768 + 176 + 2 = (946)_{10}$$

(iii)

$$\underbrace{0\,0\,1}_{1}\;\underbrace{0\,0\,1}_{1}\;\underbrace{0\,0\,1}_{1}\;\underbrace{1\,0\,1}_{5}\;\underbrace{0\,1\,0}_{2}\;\underbrace{0\,0\,1}_{1}$$

$$= (111521)_8$$

and

$$\underbrace{1\;0\;0\;1}_{9}\;\underbrace{0\;0\;1\;1}_{3}\;\underbrace{0\;1\;0\;1}_{5}\;\underbrace{0\;0\;0\;1}_{1}$$

$$= (9351)_{16}$$

Q.2 Show the value of all bits of a 12-bit register that holds the number equivalent to decimal 215 in
(a) binary (b) binary coded octal (c) binary coded hexadecimal and (d) binary coded decimal.

Solution:

(a) Binary

$$(215)_{10} = (11010111)_2$$

2	215	
2	107	1
2	53	1
2	26	1
2	13	0
2	6	1
2	3	0
	1	1

In a 12-bit register, it will be stored as: "0 0 0 0 1 1 0 1 0 1 1 1"

(b) Binary Coded Octal

$$(215)_{10} = (0327)_8 = 000\;\;011\;\;010\;\;111$$

(c) Binary Coded Hexadecimal

$$(215)_{10} = (0D7)_{16} = 0000\;\;1101\;\;0111$$

(d) Binary Coded Decimal

In binary coded decimal, each decimal (0 to 9) digit is represented by 4-bit binary code.

$$(215)_{10} = 0010\;\;0001\;\;0101$$

Q.3 Consider the addition of numbers with different bases
$(x)_7 + (y)_8 + (w)_{10} + (z)_5 = (k)_9$
If $x = 36$, $y = 67$, $w = 98$ and $k = 241$, then z is

Solution:

$$(36)_7 = (27)_{10}\;;\quad (67)_8 = (55)_{10}\;;\quad (98)_{10} = (98)_{10}$$

$$(z)_5 = (z)_5$$

$$(241)_9 = (199)_{10}$$

$$(z)_5 = (199)_{10} - (27)_{10} - (55)_{10} - (98)_{10}$$

5	19	4
	3	

$$(z)_5 = (19)_{10}$$

$$(z)_5 = (34)_5$$

$$\therefore \quad z = 34$$

Q.4 (a) Represent the 8620 into following codes:

(i) BCD (ii) Excess-3 (iii) 2421

(b) Find 7's complement of the given number $(2365)_7$

Solution:

(a) (i) Write binary equivalent of each decimal

8620 $\Rightarrow$ 1000 0110 0010 0000

(ii) **Excess-3:** For excess 3, add 3 (binary 0011) to each BCD part.

Hence,

1000	0110	0010	0000
+0011	+0011	+0011	+0011
1011	1001	0101	0011

(iii) 2421: It is a weighted binary code

There codes are minor image from the given dotted line.

As $(4)_{10}$ and $(5)_{10}$ make complementary pair.

Similarly $(3)_{10}$ and $(6)_{10}$ .... make the complementary pair.

Hence, 1110 1100 0010 0000.

Decimal digit	2	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	0	1
8	1	1	1	0
9	1	1	1	1

(b) For a value/number having a base of r,

then r's complement = $(r-1)$'s complement + 1

Hence, 7's complement of $(2365)_7$ = 6's complement + 1

$$\begin{array}{rl} 6\,6\,6\,6 & \\ -\,2\,3\,6\,5 & \\ \hline 4\,3\,0\,1 & \text{6's complement} \\ +\,1 & \\ \hline 4\,3\,0\,2 & \text{7's complement} \end{array}$$

Q.5 Perform the following conversions:

(i) $(3287.5100098)_{10}$ into octal (ii) $(675.625)_{10}$ into hexadecimal (iii) $(A72E)_{16}$ into octal

Solution:

(i) To convert $(3287.5100098)_{10}$ into octal:

- Integer part conversion,

$$\begin{array}{r|l} 8 & 3287 \\ \hline 8 & 410 - 7 \\ \hline 8 & 51 - 2 \\ \hline 8 & 6 - 3 \\ \hline & 0 - 6 \end{array} \uparrow$$

$$(3287)_{10} = (6327)_8$$

- Fractional part conversion,

$$0.5100098 \times 8 = 4.0800784 \rightarrow 4$$
$$0.0800784 \times 8 = 0.6406272 \rightarrow 0$$
$$0.6406272 \times 8 = 5.1250176 \rightarrow 5$$
$$0.1250176 \times 8 = 1.0001408 \rightarrow 1$$
$$(0.5100098)_{10} = (0.4051...)_8$$

So, $(3287.5100098)_{10} = (6327.4051...)_8$

(ii) To convert $(675.625)_{10}$ into hexadecimal:

- Integer part conversion,

$$\begin{array}{r|l} 16 & 675 \\ \hline 16 & 42 - 3 \\ \hline 16 & 2 - A \\ \hline & 0 - 2 \end{array} \uparrow \qquad (675)_{10} = (2A3)_{16}$$

- Fractional part conversion,

$$0.625 \times 16 = 10.000 \rightarrow A$$

$$(0.625)_{10} = (0.A)_{16}$$

So, $$(675.625)_{10} = (2A3.A)_{16}$$

(iii) To convert $(A72E)_{16}$ into octal:

- Hexadecimal to binary conversion,

$$(A72E)_{16} = (1010\,0111\,0010\,1110)_2$$

- Binary to octal conversion,

$$(1010\,0111\,0010\,1110)_2 = (001\,010\,011\,100\,101\,110)_2$$
$$= (123456)_8$$

So, $$(A72E)_{16} = (123456)_8$$

Q.6 If $X = 111.101$ and $Y = 101.110$ calculate $X + Y$ and $\left.\begin{array}{c} X - Y \\ Y - X \end{array}\right\}$ by 2's complement method.

Solution:

Given $$X = 111.101$$
$$Y = 101.110$$

Now $$X + Y = \begin{array}{r} 111.101 \\ 101.110 \\ \hline 1101.001 \\ \hline \end{array}$$

For $$X - Y = X + \text{2's complement of } Y$$
$$= 111.101 + 010.010$$

Discard the carry = (1)001.111 as number will be positive

$$= 001.111$$
$$= 1.111$$

For $$Y - X = Y + \text{2's complement of } X$$
$$= 101.110 + 000.011$$
$$= 110.001$$

∵ There is no carry generated its a negative number.

∴ Difference = – (2's complement of 110.001) = – 1.111

Q.7 Perform the following addition and subtraction of excess-3 numbers:

(i) 0100 1000 + 0101 1000 (ii) 1100 1011 – 0100 1001

Check the results obtained, by performing the above operations in decimal format.

Solution:

(i) 0100 1000 + 0101 1000 in excess-3 format:

$$\begin{array}{rr l}
0100 & 1000 & \\
(+)0101 & 1000 & \\
\hline
1001\,\textcircled{1} & 0000 & \text{There is a carry from lower nibble, which is to be propagated} \\
1 \hookleftarrow & & \\
\hline
1010 & 0000 & \text{Add "0011" to lower nibble} \\
(-)0011 & (+)0011 & \text{Subtract "0011" from higher nibble} \\
\hline
0111 & 0011 & \leftarrow \text{Final result in excess-3 format} \\
\hline
\end{array}$$

Checking the above result in decimal format:

$$0100\ 1000 \xrightarrow{\text{To 8421 BCD}} 00010101 \xrightarrow{\text{To decimal}} (15)_{10}$$

$$01011000 \xrightarrow{\text{To 8421 BCD}} 0010\ 0101 \xrightarrow{\text{To decimal}} (25)_{10}$$

$$(15)_{10} + (25)_{10} = (40)_{10}$$

$$(40)_{10} \xrightarrow{\text{To 8421 BCD}} 0100\ 0000 \xrightarrow{\text{To excess-3}} 01110011$$

(ii) 1100 1011 – 0100 1001 in excess-3 format:

$$\begin{array}{rr l}
1100 & 1011 & \\
(-)\ 0100 & 1001 & \\
\hline
1000 & 0010 & \text{Add "0011" to both the nibbles} \\
(+)\ 0011 & (+)0011 & \\
\hline
1011 & 0101 & \leftarrow \text{Final result in excess-3 format} \\
\hline
\end{array}$$

Checking the above result in decimal format:

$$1100\ 1011 \xrightarrow{\text{To 8421 BCD}} 10011000 \xrightarrow{\text{To decimal}} (98)_{10}$$

$$0100\ 1001 \xrightarrow{\text{To 8421 BCD}} 00010110 \xrightarrow{\text{To decimal}} (16)_{10}$$

$$(98)_{10} - (16)_{10} = (82)_{10}$$

$$(82)_{10} \xrightarrow{\text{To 8421 BCD}} 1000\ 0010 \xrightarrow{\text{To excess-3}} 10110101$$

Q.8 (i) Each of the following arithmetic operations is correct in atleast one number system. Calculate the minimum non-zero base for which the following operations are true.

1. $\frac{54}{4} = 13$ 2. $\sqrt{41} = 5$ 3. $\frac{302}{20} = 12.1$ 4. $3 \times 11 = 33$

(ii) Calculate the minimum non-zero base of x which satisfies the quadratic equation $x^2 - 11x + 22 = 0$, whose roots are $x = 3$ and $x = 6$.

Solution:

(i) 1. Let the base of the expression be 'x'.

thus,
$$\frac{(54)_x}{(4)_x} = (13)_x$$

$$\Rightarrow \quad \frac{5x+4}{4} = x + 3$$

$$\Rightarrow \quad 5x + 4 = 4x + 12$$

$$x = 8$$

Hence, the minimum non-zero base is equal to '8'.

2. Let the base of the expression be equal to 'x'.

$$\sqrt{(41)_x} = (5)_x$$

$$\sqrt{4x+1} = 5$$

$$4x + 1 = 25$$
$$x = 6$$

Hence, the minimum non-zero base for the expression is equal to 6.

3. Let the base of the expression be equal to x.

$$\frac{(302)_x}{(20)_x} = (12.1)_x$$
$$\frac{3x^2+2}{2x} = x+2+\frac{1}{x}$$
$$\frac{3x^2+2}{2} = x^2 + 2x + 1$$
$$x^2 - 4x = 0$$
$$x = 0, 4$$

Thus, the non-zero base is equal to 4.

4. Let the base of the expression be equal to 'x'.

$$(3)_x \times (11)_x = (33)_x$$
$$3(x + 1) = 3x + 3$$
$$3x + 3 = 3x + 3$$

Thus, the equation is valid for any value of x. So the minimum non-zero base will be equal to $x = 4$.

(ii) The given quadratic equation,

$$x^2 - 11x + 22 = 0 \quad \text{...(i)}$$

The factor of solution are 3 and 6.

Thus, the equation can also be written as

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$
$$x^2 - [(3)_b + (6)_b]x + (3)_b \times (6)_b = 0 \quad \text{...(ii)}$$

Equating equation (i) and (ii), we get,

$$(3)_b + (6)_b = (11)_b$$

and $$(3)_b \times (6)_b = (22)_b$$
$$3 + 6 = b + 1$$

∴ $$b = 8$$

∴ The minimum non-zero base = 8.

Q.9 What are error-correcting codes? For Hamming code write an expression for describing the location of possible error. Also, find out the value of 'K' for converting BCD code into Hamming code and the bit positions of the resulting Hamming code?

Solution:

- When the digital data is transmitted through the channel, the noise signal changes the original data and false output is obtained at the receiver. To correct this data, it is required to detect the error which is done by error detecting codes and the errors are corrected using error correction codes such as hamming code.
- Hamming codes are a family of linear error-correcting codes. Hamming codes can detect upto two-bit errors or correct one-bit error with detection of uncorrected errors.
- Hamming code is constructed by adding number of parity bits to each group of n-bit information or message in such a way so as to able to locate the bit position in which error occurs. Let us assume that k-parity bits p_1, p_2, p_3 ...., p_k are added to the n-bit message to form an $(n + k)$-bit code. The value of 'k' must be chosen in such a way so as to be able to describe the location of any of the $(n + k)$ possible error bit locations and also "no error" condition consequently, k must satisfy the inequality.

$$\boxed{2^k \geq n + k + 1} \quad \text{...(i)}$$

k-parity checks are performed on selected bits of each code word. Each parity check includes one of the parity bits.

In BCD code, no. of bits = n = 4

The value of k must be chosen to satisfy the equation (i) so,

$$2^k \geq 4 + k + 1$$
$$2^k \geq k + 5$$

- Therefore minimum value of k = 3. So three parity bits are attached to each of the BCD code for constructing the Hamming code.

Now it will be a 7-bit code with bit positions,

$$\begin{array}{ccccccc} p_1 & p_2 & n_1 & p_3 & n_2 & n_3 & n_4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$$

Q.10 (a) Multiply the following number converting to decimal

(i) $(67)_8$ and $(15)_8$ (ii) $(5F)_{16}$ and $(A7)_{16}$

(b) Perform the following operation and obtain the result in binary.

$$(11111111)_2 \div (5)_{10}$$

Solution:

(a) (i) $(67)_8$ and $(15)_8$

Converting into binary equivalent

```
  110 111 × 001 101
  -----------------
        1 1 0 1 1 1
    1 1 0 1 1 1 X X
  11 0 1 1 1 X
  -----------------
  10 1 1 0 0 1 0 1 1
```

(ii)
```
 5 F × A 7
     2 9 9
   3 B 6 ×
  --------
   3 D F 9
  --------
```

$$F \times 7 = (105)_{10} = 16 \times 6 + 9$$
$$5 \times 7 = (35)_{10} + (16)_{10} = (41)_{10} = 16 \times 2 + 9$$
$$F \times A = (150)_{10} = 16 \times 9 + 6$$
$$5 \times A = (50)_{10}$$
$$(50)_{10} + (9)_{10} = (59)_{10} = 16 \times 3 + B$$
$$(9)_{10} + (6)_{10} = (F)_{16}$$
$$(B)_{16} + (2)_{16} = (D)_{16}$$

(b) $(11111111) \div (5)_{10}$

Converting $(5)_{10}$ into binary

$$(5)_{10} = (101)_2$$

```
        11001
     ---------
 101 | 1111 1111
       101
     ---------
        101
        101
        -----
         00111
           101
           -----
            101
            101
           -----
             X
           -----
```

Hence $(1111\ 1111)_2 \div (5)_{10}$

$= 11001$

■■■■

SIGNALS AND SYSTEMS

CONVENTIONAL PRACTICE SETS

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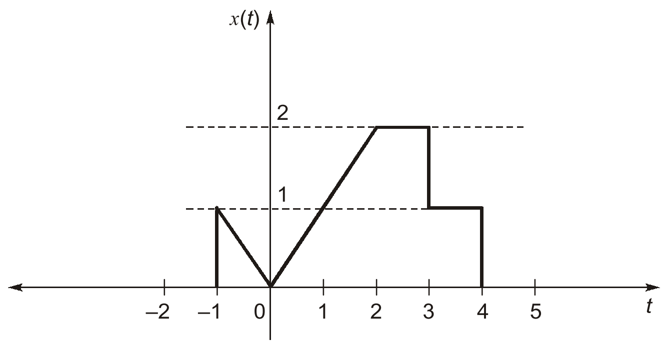
1 CHAPTER Continuous Time Signal & System

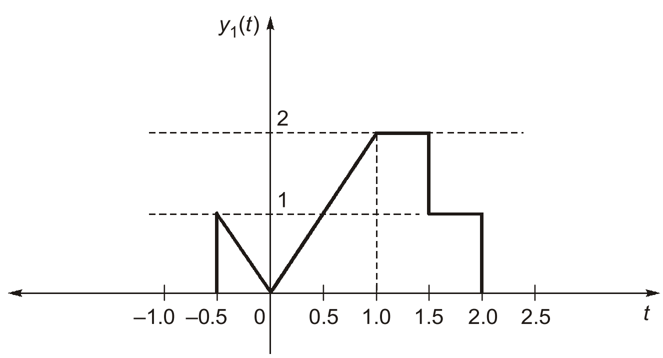
Q.1 For the given signal $x(t)$ as shown below, sketch the following signals.



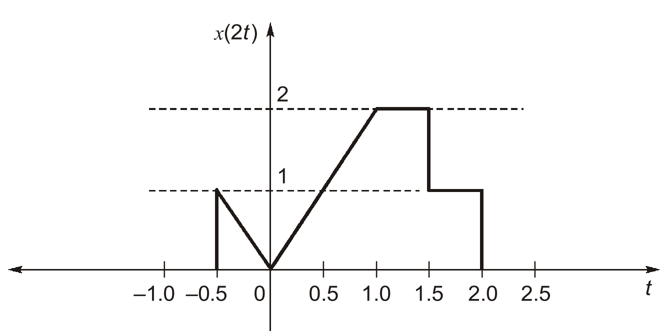
(a) $y_1(t) = x(2t)$ (b) $y_2(t) = x(2t + 4)$ (c) $y_3(t) = x\left(\frac{t}{2} + 2\right)$

Solution:

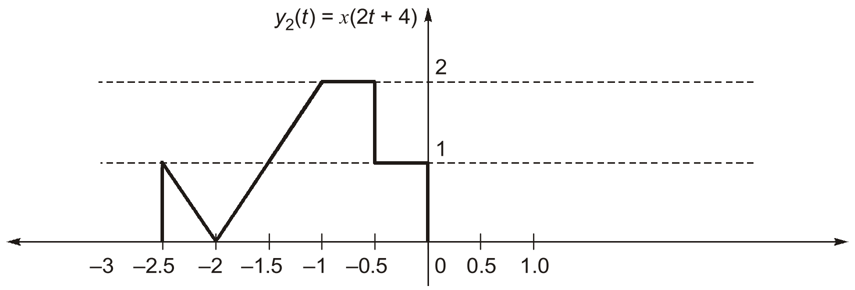
(a) We have to sketch, $y_1(t) = x(2t)$ since, $y_1(t)$ is a 2 times slowed or compressed version of $x(t)$ in time domain. So, the curve is;



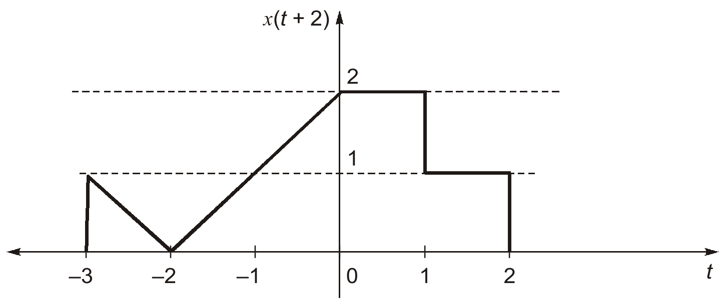
(b) We have to sketch, $y_2(t) = x(2t + 4)$ i.e. $y_2(t) = x[2(t + 2)]$.
So, we can say $y_2(t)$ is the 2 times compressed version in time of a signal which is an advance shift of 2 unit of $x(t)$.
At first we sketch the following:



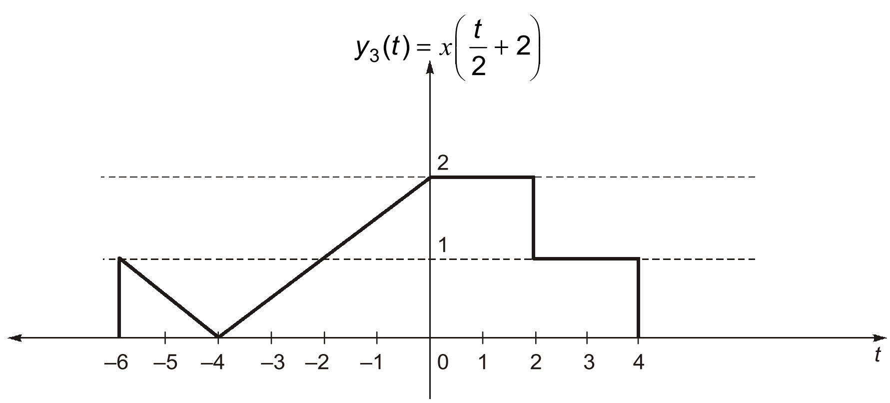
Again, we have to sketch this $x(2t)$ for an advance shift of 2 units means shift the above curve 2 unit in left side as below:



(c) We have to sketch, $y_3(t) = x(t/2 + 2)$. For this firstly, we shift 2 unit in advance shift of $x(t)$ and then expanded this signal $x(t + 2)$, by $1/(1/2) = 2$ units of the signal.



Now finally we have to sketch, $y_3(t) = x\left(\frac{t}{2}+2\right)$ as below:



Q.2 For the signals $y_1(t)$ and $y_2(t)$ shown below. Draw the differentiation of the signals and find the equations of differentiated signals.

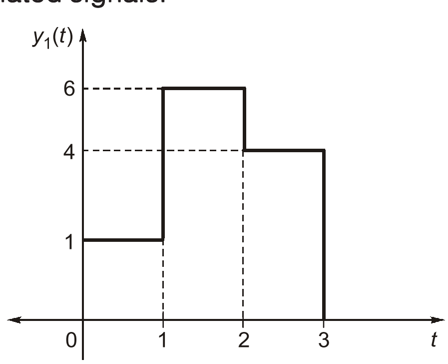


Fig. (a)

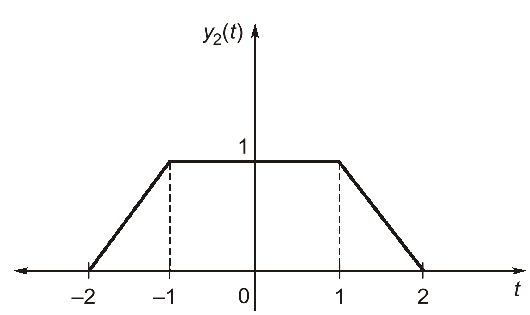


Fig. (b)

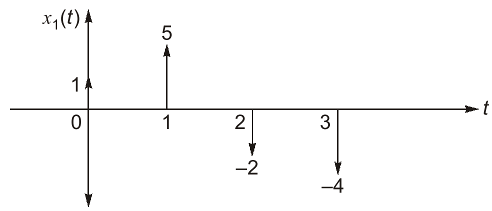
Solution:

For figure (a), we have to differentiated the signal $y_1(t)$.

Let, $$\frac{d\,y_1(t)}{dt} = x_1(t) \quad \text{...(i)}$$

Given, $$y_1(t) = u(t) + 5u(t-1) - 2u(t-2) - 4u(t-3)$$

So, from equation (i), $$x_1(t) = \frac{d\,y_1(t)}{dt} = \delta(t) + 5\delta(t-1) - 2\delta(t-2) - 4\delta(t-3)$$



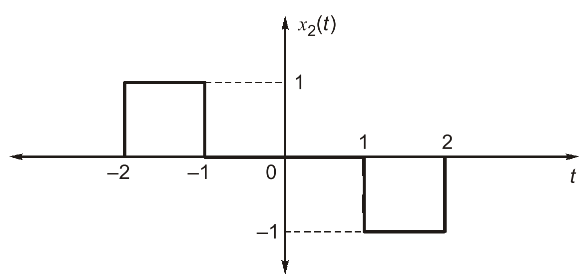
For figure (b), we have to differentiate the signal $y_2(t)$.

Let, $$\frac{d\,y_2(t)}{dt} = x_2(t) \quad \text{...(ii)}$$

Given, $$y_2(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$

(where $r(t)$ represents the ramp function)

So, from equation (ii), $$x_2(t) = \frac{d\,y_2(t)}{dt} = u(t+2) - u(t+1) - u(t-1) + u(t-2)$$



Q.3 Show that the signal, $S(t) = t^{-1/4}\,u(t-1)$ is neither an energy nor a power signal.

Solution:

For an arbitrary continuous-time signal $s(t)$, the normalized energy content 'E' of $S(t)$ is defined as,

$$E = \int_{-\infty}^{\infty} |S(t)|^2\,dt \quad \text{...(i)}$$

or $$E = \int_{-\infty}^{\infty} \left|t^{-1/4}u(t-1)\right|^2 dt$$

Since, $$u(t-1) = \begin{cases} 1, & t > 1 \\ 0, & t < 1 \end{cases}$$

$\therefore$ $$E = \int_{1}^{\infty} \left|t^{-1/4}\right|^2 dt = \int_{1}^{\infty} [t]^{-1/2}\,dt = \left[2[t]^{1/2}\right]_1^{\infty}$$

$\therefore$ $$E = \infty$$

Now, the normalized average power 'P' of $S(t)$ is defined as,

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |S(t)|^2\,dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left|t^{-1/4}u(t-1)\right|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_1^T (t^{-1/4})^2\, dt = \lim_{T\to\infty} \frac{1}{2T}\int_1^T t^{-1/2}\, dt$$

$$\therefore \quad P = \lim_{T\to\infty} \frac{1}{2T}\left[2t^{1/2}\right]_1^T = \lim_{T\to\infty}\left[\frac{T^{1/2}-1}{T}\right] = \lim_{T\to\infty}\left[\frac{1-1/\sqrt{T}}{\sqrt{T}}\right] = 0$$

$$\therefore \quad P = 0$$

Here average power $P \to 0$, when total energy $E \to \infty$, which means that the condition $0 < E < \infty$ is not satisfied. Hence signal, $S(t) = t^{-1/4}\, u(t-1)$ is not an energy signal. Also, when $E \to \infty$, the value of $P \to 0$ which means that the condition $0 < P < \infty$ is not satisfied. Therefore, signal $S(t) = t^{-1/4}\, u(t-1)$ is not a power signal.

Thus, we can say that the given signal, $S(t) = t^{-1/4}\, u(t-1)$ is neither an energy signal nor a power signal. **Proved.**

Q.4 Consider a continuous-time system with input $x(t)$ and output $y(t)$ given by

$$y(t) = x(t)\cos(t)$$

Check whether the is

(a) linear (b) time-invariant

Solution:

$$y(t) = x(t)\cos(t)$$

(a) To check linearity,

$$y_1(t) = x_1(t)\cos(t) \qquad [y_1(t) \text{ is output for } x_1(t)]$$

$$y_2(t) = x_2(t)\cos(t) \qquad [y_2(t) \text{ is output for } x_2(t)]$$

So the output for $(x_1(t) + x_2(t))$ will be

$$y(t) = [x_1(t) + x_2(t)]\cos(t)$$

$$= y_1(t) + y_2(t)$$

So the system is linear to check time invariance.

(b) The delayed output, $y(t - t_0) = x(t - t_0)\cos(t - t_0)$

The output for delayed input,

$$y(t, t_0) = x(t - t_0)\cos(t)$$

Since, $y(t - t_0) \neq y(t, t_0)$

System is time varying.

Q.5 Find out whether the system is stable/causal. If the impulse response is given by, $h(t) = e^{-6|t|}$.

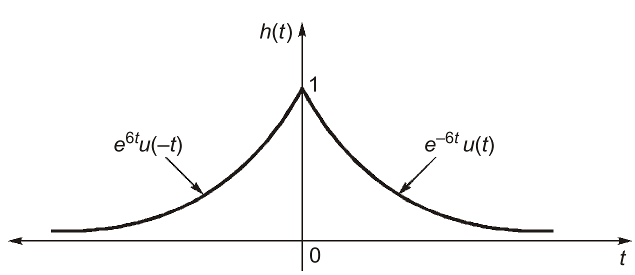
Solution:

As given that,

$$h(t) = e^{-6|t|}$$

$$\therefore \quad h(t) = e^{-6t}\cdot u(t) + e^{6t}\cdot u(-t) \qquad \text{...(i)}$$

Here we see that, $h(t) \neq 0$ for $t < 0$ so, the given system is not **causal**.



For checking the system to be **"BIBO stable"**, we know that,

$$\sum_{t=-\infty}^{\infty} h(\tau)\, d\tau < \infty \qquad \text{...(ii)}$$

L.H.S of equation (ii) = $\sum_{t=-\infty}^{\infty} h(\tau)\,d\tau$

$$= \sum_{t=-\infty}^{\infty}\left[e^{-6\tau}u(t)\,d\tau + e^{6\tau}\,u(-\tau)\,d\tau\right] = \int_{t=0}^{\infty} e^{-6\tau}\,d\tau + \int_{-\infty}^{0} e^{6\tau}\,d\tau$$

$$= -\frac{1}{6}\left[e^{-6\tau}\right]_0^{\infty} + \frac{1}{6}\left[e^{6\tau}\right]_{-\infty}^{0} = -\frac{1}{6}(0-1) + \frac{1}{6}(1-0)$$

L.H.S of equation (ii) = $\frac{1}{3} < \infty$

So, the system is "BIBO Stable".

Q.6 A continuous time LTI system is described by:

$$y(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau)\,d\tau$$

Find the impulse response of the system. Is the system casual?

Solution:

1st Method:

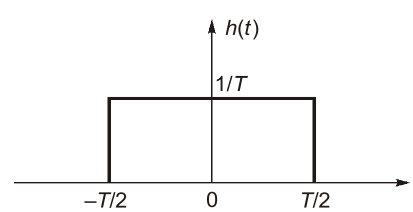
$$y(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(\tau)\,d\tau \qquad \text{...(i)}$$

Let the impulse response be $h(t)$,

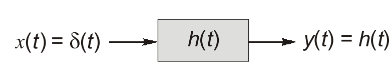
$$\therefore \quad y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \qquad \text{(ii)}$$

Comparing equation (i) and (ii) we get,

$$h(t) = \begin{cases} \frac{1}{T} & \frac{-T}{2} < t < \frac{T}{2} \\ 0 & \text{Otherwise} \end{cases}$$



2nd method:



$$\therefore \quad h(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \delta(t)dt$$

$$\Rightarrow \quad h(t) = u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)$$

The system is not causal as we can clearly see from equation (i) that for calculation of $y(t)$ at any time t, we require future values of input $x(t)$.

Another way to see this is that $h(t)$ is not zero for $t < 0$, which is the basic requirement for any causal system.

Q.7 Show that the following properties holds good for the derivative of $\delta(t)$.

(a) $\int_{-\infty}^{\infty} \phi(t)\,\delta'(t)\,dt = -\phi'(0)$

where, $\phi'(0) = \left.\frac{d\phi(t)}{dt}\right|_{t=0}$

(b) $t\delta'(t) = -\delta(t)$